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A NEW CLASS OF CONTROL USING FRACTAL PROCESSES

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ABSTRACT
In the past decade, control of dynamical systems with classical and advanced controls such as optimal control, adaptive control and Fuzzy logic. These controls have been investigated extensively to identify which is the appropriate control to use during the dynamical processes of mitosis and meiosis in biology processes.However, the above controls use a fractal control which cannot be used, our investigation has identified that a classic approach is needed. This paper provides a new approach and may be the first in systematically dealing with fractal controls. One of the most important aspects of model cell biology is the understanding of complicated dynamical processes that take place such as mitosis, meiosis and duplication. This paper presents the methods used for modeling and controlling a biology process such as mitosis, meiosis and duplication. This approach is inspired from the Julia process. Simulation results have illustrated the effectiveness of the proposed schemes.

Keywords
Fractal, control, mitosis, process, Julia process

1. INTRODUCTION
Fractal processes are used extensively for modeling different processes in Nature as well as for identifying biology processes. Many analysis and synthesis classes for fractal processes have been proposed, such as Iterated Function Systems, Iterated Complex Functions, Linder Mayer Systems, and Geometry objects, etc....

In recent years, studies have shown that embedded in a Julia set there hides an intricate regular structure [1], thus the fractal theory is enriched[2],[3]. The many approaches introduced to construct the Julia set has created a platform for extensive research and experiment. It was the introduction of Gaston Julia's methods which has allowed scholars such as Lahtakia to identify methods of constructing a switched Julia set. Also, Michelistsch and *Rö*ssler developed a method of constructing the Julia set using a simple non analytic complex mapping. Alan Norton managed to display the spatial Julia sets in 4-D quaternion, using a boundary tracking method [4]. Ping Liu et al identified how to control and synchronize a Julia set in a coupled map lattice using gradient and optimal controls[5]. Pickover, Hooper and Philip studied the non boundary region fractal structure of Mandelbrot set. In this paper, an approach is proposed for a new class of controls using fractal processes based on an iterated complex function inspired from Julia process. Different methods for control have been developed such as: sliding mode control, backstepping control, adaptive control, bang-bang optimal control, etc.... All those methods are used for nonlinear dynamic systems unlike biology processes. Which controls do biology processes use to make replication, duplication, division, metamorphoses and multiplication? also this paper will explain the importance of biological processes by identifying whether mitosis and meiosis processes have a fractal structure.

Section 1 presents the three different algorithms used throughout this paper. In Section 2 we elaborated a fractal cell by given a non linear dynamic system based on fractal processes and transformation. Section 3 gives an overview of processes of cell division in genetical biology. Then, section 4 presents four theorems with the use of mathematical proofs. Section 5 shows how some algorithms of control were implemented to simulate mitosis, duplicate and metamorphoses processes in the cell fractal model. Finally, section 6 concludes this paper.

2. FRACTAL ALGORITHM
This section shows how algorithms were inspired from Julia processes starting by explaining the iterative map of Julia.

In 1919, Gaston Julia put forward the simple iterative map[1]

$$Z_{n+1} = Z_n^2 + Z_c \tag{1}$$

The properties used to generate Julia processes are well known , these are:

- The Julia set is a repellor.
- The Julia set is invariant.
- An orbit on Julia set is either periodic or chaotic.
- All unstable periodic points are on Julia set.

- The Julia set is either wholly connected or wholly disconnected.
- All sets generated only with Julia sets combination have fractal structure[8].

We study Julia sets of polynomials of the form

$$\begin{cases} x_{i+1} = x_i^2 - y_i^2 + x_c \\ y_{i+1} = x_i y_i + y_c \end{cases} \tag{2}$$

To find the inverse map, one must find expressions for x_i and y_i in terms x_{i+1} and y_{i+1}. After computation we obtain

$$\begin{cases} x_i = \pm\sqrt{\sqrt{(x_{i+1}-x_c)^2 + (y_{i+1}-y_c)^2} + (x_{i+1}-x_c)} \\ y_i = \frac{y_{i+1}-y_c}{2x_i} \end{cases} \tag{3}$$

For all states $(x_{i+1}, y_{i+1}) = P_J(x_i, y_i)$ we modified the system above in three cases as follows:

- **PROCESS** P

  Let f, g and h mathematic at function.

$$\begin{cases} x_i = \pm g[f(x_{i+1}-x_c)^2 + (y_{i+1}-y_c)^2 + (x_{i+1}-x_c)] \\ y_i = \frac{y_{i+1}-y_c}{2x_i} \end{cases} \tag{4}$$

$$\begin{cases} y_{i+1} = h[f[(x_i)^2 + (y_i)^2] - \frac{x_i}{2}] \\ x_{i+1} = \frac{y_i}{2x_{i+1}} \end{cases} \tag{5}$$

- **PROCESS** P_1

  P_J is Julia process $(x_{i+1}, y_{i+1}) = P_J(x_i, y_i)$ we modified the input states as follows:

$$\begin{cases} u_i = \alpha x_i + \beta \\ v_i = \lambda y_i + \mu \end{cases} \tag{6}$$

  we used P_1 when we execute the process $(u_{i+1}, v_{i+1}) = P_1(u_i, v_i)$

- **PROCESS** P_2

  we used P_2 when we execute the process

$$\begin{cases} u_i = \pm\sqrt{\sqrt{(u_{i+1})^2 + (v_{i+1})^2} + (u_{i+1})} \\ v_i = \frac{v_{i+1}}{2u_i} \end{cases} \tag{7}$$

We noted P_1, P_2 and P

2.1 Algoritm of Process P

In this subsection, we present three algorithms that we will use in the this paper

Algorithm 1 $(y_{i+1}, x_{i+1}) = P(x_i - xc, y_i - yc)$

if $x_i < x_c$ **then**
  $x_{i+1} = g[f[(x_i - x_c)^2 + (y_i - y_c)^2] + \frac{x_i}{2}]$
  $y_{i+1} = \frac{y_i - y_c}{2x_{i+1}}$
end if
if $x_i = x_c$ **then**
  $x_{i+1} = \sqrt{\frac{|y_i - y_c|}{2}}$
  if $x_i > 0$ **then**
    $x_{i+1} = \frac{y_i - y_c}{2y_{i+1}}$
  end if
  if $x_i < 0$ **then**
    $y_{i+1} = 0$
  end if
end if
if $x_i > x_c$ **then**
  $y_{i+1} = h[f[(x_i - x_c)^2 + (y_i - y_c)^2] - \frac{x_i}{2}]$
  $x_{i+1} = \frac{y_i - y_c}{2x_{i+1}}$
  if $y_i < y_c$ **then**
    $y_{i+1} = -y_{i+1}$
  end if
end if

2.2 Algoritm of Process P_1

The listing of algorithm P_1 is as follows:

Algorithm 2 $(y_{i+1}, x_{i+1}) = P_1(\alpha_1 x_i + \beta_1, \alpha_2 y_i + \beta_2)$

if $\alpha_1 x_i < +\beta_1$ **then**
  $x_{i+1} = \sqrt{\sqrt{(\alpha_1 x_i + \beta_1)^2 + (\alpha_2 y_i + \beta_2)^2} + \frac{\alpha_1 x_i + \beta_1}{2}}$
  $y_{i+1} = \frac{\alpha_2 y_i + \beta_2}{2x_{i+1}}$
end if
if $\alpha_1 x_i = +\beta_1$ **then**
  $x_{i+1} = \sqrt{\frac{|\alpha_2 y_i + \beta_2|}{2}}$
  if $\alpha_1 x_i + \beta_1 > 0$ **then**
    $x_{i+1} = \frac{\alpha_2 y_i + \beta_2}{2y_{i+1}}$
  end if
  if $\alpha_1 x_i + \beta_1 < 0$ **then**
    $y_{i+1} = 0$
  end if
end if
if $\alpha_1 x_i > +\beta_1$ **then**
  $y_{i+1} = \sqrt{\sqrt{(\alpha_1 x_i + \beta_1)^2 + (\alpha_2 y_i + \beta_2)^2} - \frac{\alpha_1 x_i + \beta_1}{2}}$
  $x_{i+1} = \frac{\alpha_2 y_i + \beta_2}{2x_{i+1}}$
  if $\alpha_2 y_i < +\beta_2$ **then**
    $y_{i+1} = -y_{i+1}$
  end if
end if

2.3 Algoritm of Process P_2

Algorithm 3 $(y_{i+1}, x_{i+1}) = P_2(x_i, y_i)$

```
if x_i < 0 then
    x_{i+1} = sqrt( sqrt((x_i)^2 + (y_i)^2) + x_i/2 )
    y_{i+1} = y_i / (2 x_{i+1})
end if
if x_i = 0 then
    x_{i+1} = sqrt( |y_i| / 2 )
    if x_i > 0 then
        x_{i+1} = y_i / (2 y_{i+1})
    end if
    if x_i < 0 then
        y_{i+1} = 0
    end if
end if
if x_i > 0 then
    y_{i+1} = sqrt( sqrt((x_i)^2 + (y_i)^2) - x_i/2 )
    x_{i+1} = y_i / (2 x_{i+1})
    if y_i < 0 then
        y_{i+1} = -y_{i+1}
    end if
end if
```

In this paper, we show how the biology process on fractals can be described and modeled.

3. MATHEMATIC FORMULATION OF FRACTAL CELL

We form a fractal cell by combining fractal processes with transformation.

Let $\mathcal{E}$ be the complete metric unit, Φ a system of nonlinear dynamic processes of $\mathcal{E}$ in $\mathcal{E}$ such as:

$$\mathcal{E} \to \mathcal{E}$$

$$\Phi{:}(x_i, y_i) \to (x_m, y_m)$$

The system of nonlinear dynamic processes recurrent iterative Φ is represented by:

$$\Phi \begin{cases} (x_0, y_0) \\ (x_{1,1}, y_{1,1}) = P_1(\alpha x_0 + \gamma, \beta y_0 + \lambda) \\ (x_{2,2}, y_{2,2}) = P_2(x_{1,1}, y_{1,1}) \\ \vdots \\ (x_{i+1,T_1}, y_{i+1,T_1}) = T_1(x_{i,j-1}, y_{i,j-1}) \\ (x_{i+2,j}, y_{i+2,j}) = P_j(x_{i+1,T_1}, y_{i+1,T_1}) \\ \vdots \\ (x_{i+2,n}, y_{i+2,n}) = T_k(x_{i+1,n-1}, y_{i+1,n-1}) \\ \vdots \\ (x_{i+1,m}, y_{i+1,m}) = P_m(x_{i,m-1}, y_{i,m-1}) \end{cases} \quad (8)$$

The dynamics of the fractal cell is managed by the assignment of x_{i+1} in x_i and of y_{i+1} in y_i.

$$\begin{cases} x_i \leftarrow x_{i+1} \\ y_i \leftarrow y_{i+1} \end{cases} \quad (9)$$

The system (8) is a combination of different transformations and different processes. It consists of m equations and k changes, so $m - k$ process for n iterations where the first iteration is (x_0, y_0).

3.1 Model of fractal process

In this part one presents a simple example of a fractal process generated by a combination of algorithm P_1.

3.2 Model 1

Let Φ_1 a fractal process defined by:

$$\mathcal{E} \to \mathcal{E}$$

$$\Phi_1{:}(x_i, y_i) \to (x_{i+4}, y_{i+4})$$

with

$$\Phi_1 \begin{cases} (x_{i+1}, y_{i+1}) = P_1(x_i - a, y_i - b) \\ (x_{i+2}, y_{i+2}) = P_1(x_{i+1} - a, y_{i+1} - b) \\ (x_{i+3}, y_{i+3}) = P_1(x_{i+2} - a, y_{i+2} - b) \\ (x_{i+4}, y_{i+4}) = P_1(-x_{i+3} + x_{i+2}, -y_{i+3} + y_{i+2}) \end{cases} \quad (10)$$

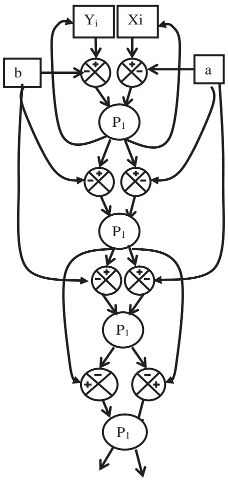
The dynamics of the cell Φ_1 is managed by:

$$\begin{cases} x_i \leftarrow x_{i+1} \\ y_i \leftarrow y_{i+1} \end{cases} \quad (11)$$

The model given by the system 10 comprises a process which repeats itself in four iterations of levels .

Being given an initial state of couple (x_0, y_0) and a couple of constant (a, b).

The first iteration begins with the processing of P_1 state vector $(x_0 - a, y_0 - b)$ to provide an output of state (x_1, y_1). On the second level of iteration, the P_1 process treats the state vector $(x_1 - a, y_1 - b)$ for outputting a pair of 'state (x_2, y_2). The previous couple is compared with the condition of constant (a, b) that becomes a state vector for a coordinated $(x_2 - a, y_2 - b)$, this result is treated by P_1 in the third level iteration to give a state vector (x_3, y_3). The treatment of P_1 $(x_2 - x_3, y_2 - y_3)$ gives the vector (x_4, y_4). The behavior of the output vector (x_{i+4}, y_{i+4}) for i iteration is shown in Figure 1(b).



(a) The graph

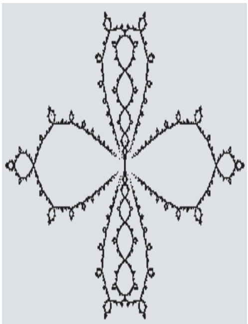

(b) Behavior of cell process

Figure 1: A fractal cell

4. PROCESSES OF CELL DIVISION

This section presents the processes of reproduction and growth by which the cells become divided into daughter cells.

1. **Duplication**

   A type of mutation in which a portion of a genetic material or a chromosome is duplicated or replicated, resulting in multiple copies of that region.

2. **Mitosis**

   Mitosis is a form of eukaryotic cell division that produces two daughter cells with the same genetic components as the parent cell. Mitosis consists of continuous processes, which are conventionally divided into five stages: prophase, prometaphase, metaphase, anaphase and telophase.

3. **Meiosis**

   Mitosis creates two identical daughter cells that each contain the same number of chromosomes as their parent cell. In contrast, meiosis gives rise to four unique daughter cells, each with half the number of chromosomes as the parent cell. Because meiosis creates cells that are destined to become gametes (or reproductive cells), this reduction in the number of chromosomes is essential - without it, the union of two gametes during fertilization would result in offspring with twice the normal number of chromosomes. Like mitosis, meiosis also has distinct stages called prophase, metaphase, anaphase and telophase. A key difference, however, is that during meiosis, each of these phases occurs twice - once during the first round of division, called meiosis I and again in the second round of division, called meiosis II .

4. **Metamorphosis**

   The change in the form or function and behaviour of a living organism, by a natural process of growth or development such as, the metamorphosis of the yolk into the embryo, of a tadpole into a frog, or of a bud into a blossom.

5. MATHEMATIC FORMULATION OF CONTROLS PROCESSES

5.1 Process control of duplication

P is a process applied in cascade with P_2 states whose outputs P are inputs and the outputs of P_2 will be reinfected into the process P. The output state of P_2 are elements of P after k iterations, we obtain in terms of phase two processes identical to P.

THEOREM 5.1.1. *Let P and P_2 are two nonlinear dynamic processes. If P is applied in cascade to n processes P_2, so it was the output of nth elements of P $2n$ process duplicated*

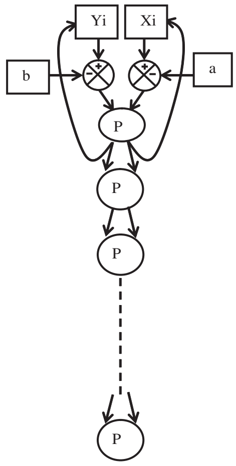


Figure 2: Graph of duplication of process control

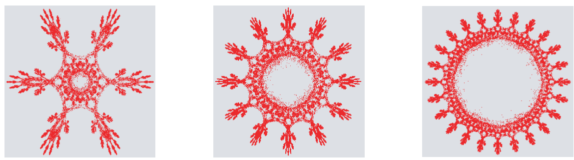


Figure 3: Duplication of number of leaf

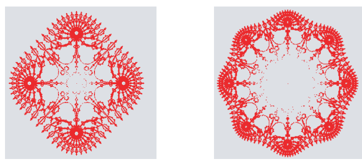


Figure 4: Second example of duplication

5.2 Process control of division

THEOREM 5.2.1. *Let P_j and P_1 two fractal processes, then one has :*

i) *If we have P_j and P_1 in cascades After treatment of (x_i, y_i) by P_j, we obtain two output states (x_{i+1}, y_{i+1}) such that $P_j(x_i, y_i) = (x_{i+1}, y_{i+1})$*

ii) ***There exists*** *β_1 et $\beta_2 \in \mathbb{R}$ such that the treatment by P_1 of $(x_{i+1} + \beta_1, y_{i+1} + \beta_2)$ divides P_j in two processes identical similar as P_j.*

Figure 6 represents the implementation of the model **Graph of model**:

Figure 8 shows graph of model of a division process

$\alpha_1 = K_1$, $\beta_1 = L_1$, $\alpha_2 = K_2$ and $\beta_2 = L_2$

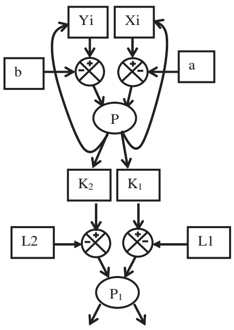
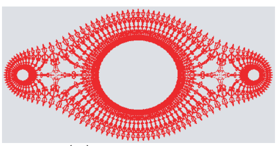


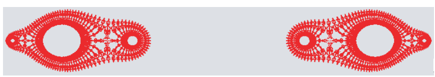
Figure 5: Graph of dividing process

Figure 6 shows the implementation

- First example



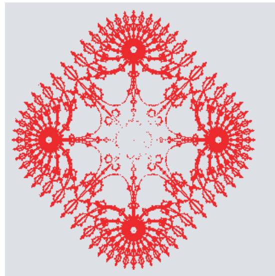
(a) Fractal cell



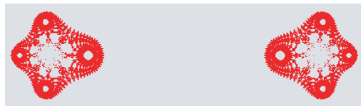
(b) Two fractals cells

Figure 6: Control process divided the fractal cell in two fractal similar

- Second example



(a) Other pattern of mother cell



(b) Two daughter cells

Figure 7: Two daughter identical cells

5.3 Multiplication process

THEOREM 5.3.1. *Let P, P_1 and P_2 three dynamical numerical non linear processes, then one has :*

i) *After treatment of (x_i, y_i) by P, we obtain two output states (x_{i+1}, y_{i+1})such that $P(x_i, y_i) = (x_{i+1}, y_{i+1})$*

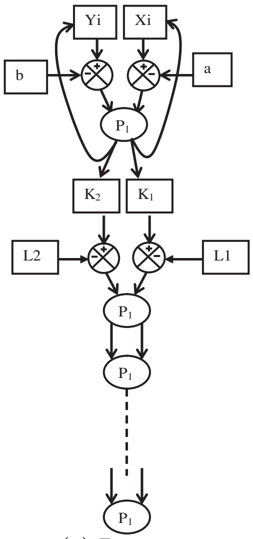
ii) ***There exists*** *α_1, α_2, β_1 et $\beta_2 \in \mathbb{R}$ such that the treatment by P_1 of $(\alpha_1 x_{i+1} + \beta_1, \alpha_2 y_{i+1} + \beta_2)$ gives two output states x_{i+2} and y_{i+2}.*

iii) *If we have n processes P_2 in cascades, then the behavior of n^{th} process is equal to 2^n behavior of states x_{i+2} and y_{i+2}. Moreover, the behavior at the end of n processes is a form with a circular distribution of angles equal to $\frac{\pi}{2^n}$.*

Proof:

Using the preceding result and the contraction principle, we obtain the fundamental result of E. de Amo [6] and Hutchinson [7]

Figure 8 shows a model of the multiplication graph.



(a) Processus

Figure 8: Graph of separation and multiplication

Simulation of process of multiplication

In some cases we validate the numerical implementation of the theorem. Figure 10 shows the behavior

1. First example

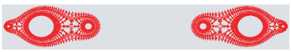


Figure 9: Cells from separation process

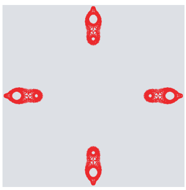


Figure 10: 2 times multiplication(duplication)

2. Second example

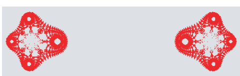


Figure 11: Process after separation

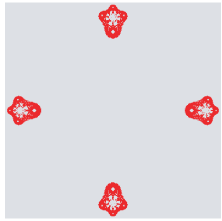


Figure 12: Multiplication by 2

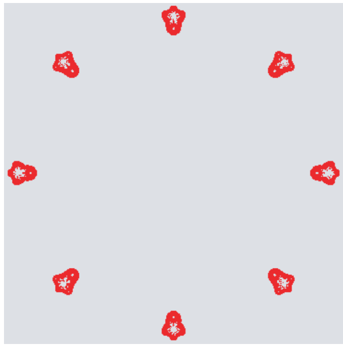


Figure 13: Multiplication by 4 in circle

The proliferation of cells is strongly linked to the number of processes in cascades. These cells are distributed in circular forms. The angle of cells is repertited equal to $\frac{\pi}{n}$.

The antecedent process has the same behavior as the process mother the only difference is on the level of dimension. The copy is identical to the level of form and behavior.

5.4 Separation process

THEOREM 5.4.1. *Let P and P_1 two nonlinear dynamic processes.*

Then:

i) *Treatment by P of the two input x_i and y_i gives in two output states x_{i+1} and y_{i+1}.*

ii) ***There exists*** *two continuations U_n and V_n double dimensional, of respective coefficients (α_n, β_n) and $((\lambda_n, \mu_n))$ such as the treatment by P_1 from $(\alpha_n x_{i+1}+\beta_n, \lambda_n y_{i+1}+\mu_n)$, gives at exit the different phases of separation of the process P into two identical processes.*

Figure illustrates a graph of separation control process.

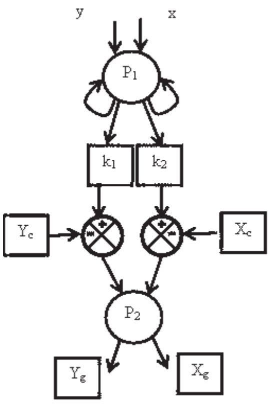


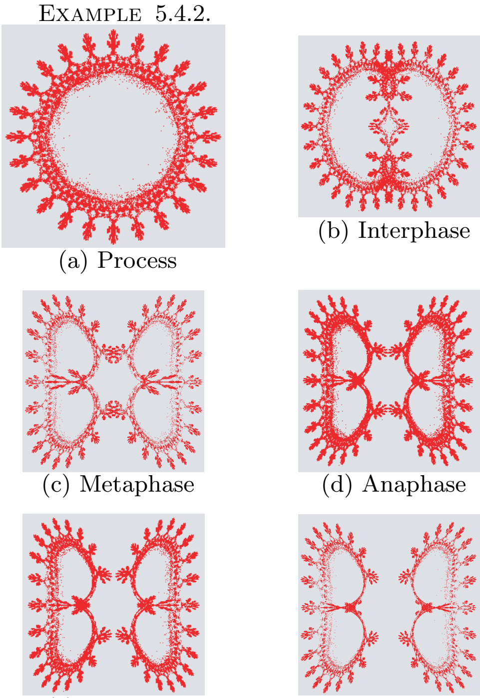
Figure 14: Graph of separation control process

Simulation of separation process In the statement of the preceding theorem the two continuations considered (α_n, β_n) and $((\lambda_n, \mu_n))$ make it possible to illustrate the evolution of process of separation of the course of time.

The results obtained show that the mother cell goes through the following phases:

prophase, anaphase, metaphase, telophase and then separates into two identical cells, so it could be a similar process in genetic biology of mitosis.

EXAMPLE 5.4.2.



(a) Process (b) Interphase (c) Metaphase (d) Anaphase (e) Telophase (f) Two daughter cells

Figure 15: Mitosis

Figure 15 illustrates the process of different phases from separation from the fractal cell into two daughter cells, which is in agreement with the process of figure 16 which shows the different phases of the cellular division.

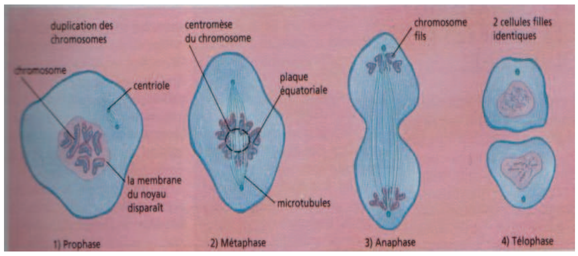
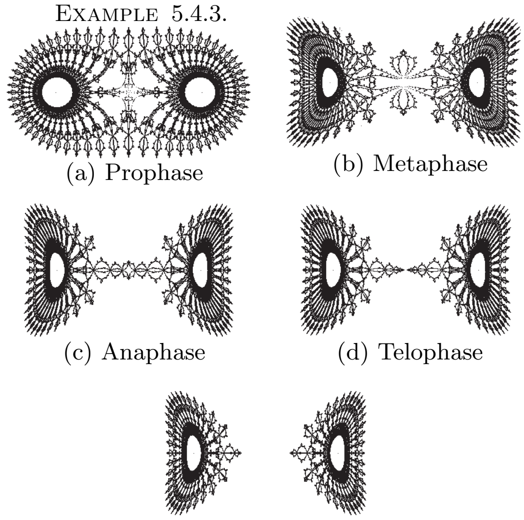


Figure 16: Process of mitosis in cell

The second example show a different stage of mitosis process

EXAMPLE 5.4.3.



(a) Prophase (b) Metaphase (c) Anaphase (d) Telophase (e) Two daughters cells

Figure 17: Fractal cell in mitosis

The separation process is a process model of mitosis, which is a continuous process, which is conventionally divided into five stages: prophase, prometaphase, metaphase, anaphase and telophase.

- Prophase: The figures 15(b) and 17(b) illustrate a behavior similar to the phase of prophase.
- Prometaphase The figures 15(c) and 17(c) illustrate a behavior similar to the phase of prometaphase.
- Metaphase The figures 15(d) and 17(d) illustrate a behavior similar to the phase metaphase.
- Anaphase The figures 15(e) and 17(e) illustrate a behavior similar to the phase anaphase.
- Telophase The figures 15(f) and 17(e) illustrate a behavior similar to the phase of telophase.

5.5 Metamorphosis

The change in the body and object structures in a state of equilibrium i to a stable state which is a more stable equilibrium $i+1$ is a cycle of metamorphosis.

If the structure of a body or object does not change from state i to the state $i+1$, we say that the body or object is at equilibrium or the absolute total convergence, then this is the end of the cycle of metamorphosis.

We define the metamorphosis as the evolution of an object from one cycle to cycle until full convergence.

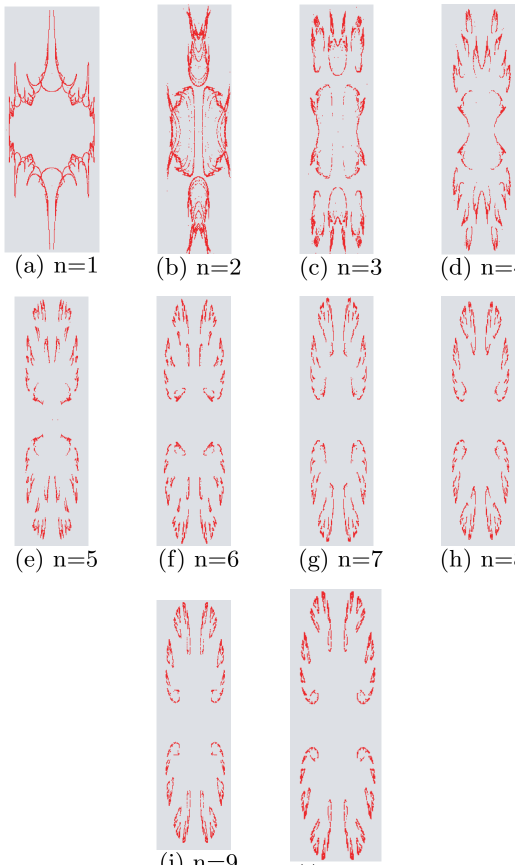
Definition 5.5.1. *Let P be a dynamic process. Suppose that there exists $n \in \mathbb{N} \setminus \{0,1\}$ such that the association result of P in n cascades is identical to that of P in $(n+1)$ cascades. Then a such n is called the convergence order of P.*

Theorem 5.5.2. *There exists a unique $A \in \mathbb{H}(X)$ such that $P^n(A) = A$. The set A is called the attractor of the process P[6]*

The following table shows the number of cascade process P_5.

Process	n cascades	Figure:
P_5	$n=2$	Fig: 18(b)
P_5	$n=3$	Fig: 18(c)
P_5	$n=4$	Fig: 18(d)
P_5	$n=5$	Fig: 18(e)
P_5	$n=6$	Fig: 18(f)
P_5	$n=7$	Fig: 18(g)
P_5	$n=8$	Fig: 18(h)
P_5	$n=9$	Fig: 18(i)
P_5	$n=10$	Fig: 18(j)

Table 1: Metamorphosis of cell generated by process P_5



(a) n=1 (b) n=2 (c) n=3 (d) n=4
(e) n=5 (f) n=6 (g) n=7 (h) n=8
(i) n=9 (j) n=10

Figure 18: Metamorphose process of cell generated by P_5

Figure 18 shows the evolution process P_5. For all $n > 10$the P_5 process shows the same behavior as for $n = 10$. The metamorphosis process P_5 for convergence order $n = 10$. Figure 18(j) is the attractor process P_5.

6. CONCLUSION

The results of this work demonstrate the capacity of fractal theory to model processes in biology. We have developed a new class of control applied in biology such as cell division processes.

The following conclusions were derived from the current work:

- We have elaborated a new methodology of generating a fractal cell.
- We started by giving an overview of processes of cell division in genetic biology.
- We have presented four control fractal processes such as duplication, multiplication, separation and divide process.
- We have implemented algorithms to prove our approach with examples.

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